

第五届“偏微分方程及其应用”学术论坛



华中师范大学数学与统计学学院

湖北 武汉

2020年5月23日至24日

第五届“偏微分方程及其应用”学术论坛

学术委员会:

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韩青、黄勇、蒋美跃、江松、李从明、李万同、刘兆理、麻希南、
王志强、辛周平、杨健夫、杨孝平、张立群、张志涛、钟承奎、
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◆会议网站：<http://maths.ccnu.edu.cn/2020pde/index.html>

◆会议平台：腾讯会议（相关会议号与密码, 详见上面网站）

◆学术报告：5月23日（周六）全天、5月24日（周日）上午

◆会议联系人：李奇：15623103286, email: qili@mails.ccnu.edu.cn

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学术报告安排表

日期: 5月23日

主持人	时间	报告人, 题目	地点
邓引斌	8:30 9:00	开幕式	腾讯会议
辛周平	9:00 9:40	刘兆理, 首都师范大学 Normalized solutions for a class of nonlinear Choquard equations	腾讯会议
江松	9:40 10:20	保继光, 北京师范大学 Asymptotic behavior at infinity of solutions of Lagrangian mean curvature equation	腾讯会议
	10:20-10:40	间 歇	
朱熹平	10:40 11:20	陈化, 武汉大学 Estimates of Dirichlet eigenvalues for fractional Laplacian	腾讯会议
钟承奎	11:20 12:00	张立群, 中科院数学与系统科学研究院 On the steady Prandtl boundary layer expansions	腾讯会议
午休			

学术报告安排表

日期: 5 月 23 日

主持人	时间	报告人, 题目	地点
麻希南	14:20 15:00	王志强, 美国Utah州立大学 Coupled nonlinear Schrödinger equations with mixed coupling	腾讯会议
曹道民	15:00 15:40	邹文明, 清华大学 Positive least energy solutions for k-coupled Schrödinger system with critical exponent: the higher dimension and cooperative case	腾讯会议
杨健夫	15:40 16:20	张志涛, 中科院数学与系统科学研究院 Existence of solutions for Schrödinger systems with linear and nonlinear couplings	腾讯会议
	16:20-16:40	间 歇	
李万同	16:40 17:20	蒋美跃, 北京大学 Periodic and quasi-periodic solutions of 1-d q-curvature equation	腾讯会议
郭玉霞	17:20 18:00	罗鹏, 华中师范大学 The number and location of critical points of positive solutions of nonlinear elliptic equations	腾讯会议

学术报告安排表

日期: 5月24日

主持人	时间	报告人, 题目	地点
李从明	8:20 9:00	韩青, 美国Notre Dame大学 The Loewner-Nirenberg problem in cones	腾讯会议
杨孝平	9:00 9:40	丁彦恒, 中科院数学与系统科学研究院 变分法与交叉科学	腾讯会议
戴求亿	9:40 10:20	周焕松, 武汉理工大学 Eigenvalue problem for a p-Laplacian equation with trapping potentials	腾讯会议
	10:20-10:40	间 歇	
黄勇	10:40 11:20	韩丕功, 中科院数学与系统科学研究院 不可压缩Navier-Stokes方程	腾讯会议
周风	11:20 12:00	帅伟, 华中师范大学 Existence and multiplicity of solutions for logarithmic Schrödinger equations with potential	腾讯会议

报告摘要

Asymptotic behavior at infinity of solutions of Lagrangian mean curvature equation

保继光, 北京师范大学

We obtain the asymptotic behavior of solution under quadratic growth condition of a class of Lagrangian mean curvature equations $F_\tau(\lambda(D^2u)) = f(x)$ in the exterior domain, where f satisfies a given asymptotic behavior at infinity. When $f(x)$ is a constant near infinity, it is not necessary to demand the quadratic growth condition anymore. These results are a kind of exterior Liouville theorem and can also be regarded as an extension of theorems of Pogorelov, Flanders and Yuan on Monge-Ampere equations and special Lagrangian equations.

Estimates of Dirichlet eigenvalues for fractional Laplacian

陈化, 武汉大学

Let $\Omega \subset \mathbb{R}^n$ ($n \geq 2$) be a bounded domain with smooth boundary $\partial\Omega$. In this talk, we shall study the Dirichlet eigenvalue problem of the fractional Laplacian $(-\Delta)^s$ which restricted to Ω with $0 < s < 1$. Denote by λ_k the k^{th} Dirichlet eigenvalue of $(-\Delta)^s$ on Ω . Firstly, we establish the explicit upper bound estimates of the ratio λ_{k+1}/λ_1 , which have polynomially growth in with optimal increase orders. Secondly, we also give the explicit lower bounds for the Riesz mean function $R_\sigma(z) = \sum_k (z - \lambda_k)_+^\sigma$ with $\sigma \geq 1$.

变分法与交叉科学

丁彦恒, 中科院数学与系统科学研究院

变分原理是自然界事物遵守的客观法则. 变分方法随着人类认识世界及改造世界的需要而产生而发展起来. 近代临界点理论在非线性科学研究中的

突出作用充分展示了变分方法是研究自然科学、社会科学等交叉科学的十分重要的工具和手段. 本报告将举例简要介绍变分方法在交叉科学研究中的重要作用.

不可压缩Navier-Stokes方程

韩丕功, 中科院数学与系统科学研究院

本报告系统介绍千禧问题之一: 不可压缩Navier-Stokes方程相关性质. 首先介绍该方程的来源及背景, 弱解存在性、正则性等的研究历史和现状; 其次, 介绍解的长时间渐近行为性质; 最后列出一些目前尚未解决的重要问题.

The Loewner-Nirenberg problem in cones

韩青, 美国Notre Dame大学

Loewner and Nirenberg discussed complete metrics conformal to the Euclidean metric and with a constant scalar curvature in bounded domains in the Euclidean space. The conformal factors blow up on boundary. The asymptotic behaviors of the conformal factors near boundary are known in C^2 -domains. In this talk, we discuss asymptotic behaviors near vertices of cones. We will prove that solutions on finite cones are well-approximated by the solution in the corresponding infinite cone. To derive optimal estimates, we need to study a class of elliptic operators over spherical domains. These operators are singular on boundary. We will study the eigenvalue problem with the homogeneous Dirichlet boundary value and investigate boundary behaviors of the eigenfunctions.

Periodic and quasi-periodic solutions of 1-d q-curvature equation

蒋美跃, 北京大学

We consider the following 4-th order ODE

$$u^{iv} + 10u'' + 9u = \frac{9}{u^{\frac{5}{3}}}, \quad (1)$$

the quantity $Q_g = \frac{1}{9}u^{\frac{5}{3}}(u^{iv} + 10u'' + 9u)$ is called Q-curvature of (S^1, g) with $g = u^{-\frac{4}{3}}g_0$, g_0 being the standard metric and u being a positive function defined on S^1 . It is well known that for $\lambda \in (0, 1], \theta \in [0, \pi]$,

$$u_{\lambda, \theta}(x) = (\lambda^{-2} \cos^2(x - \theta) + \lambda^2 \sin^2(x - \theta))^{\frac{3}{2}}$$

are π -periodic solutions of (1), and these are the all π -periodic and 2π -periodic solutions of (1).

In this talk we will discuss other types of solutions, such as periodic and quasi-periodic solutions of (1) via elementary methods.

Normalized solutions for a class of nonlinear Choquard equations

刘兆理, 首都师范大学

We prove the existence of a least energy solution to the problem

$$-\Delta u - (I_\alpha * F(u))f(u) = \lambda u \quad \text{in } \mathbb{R}^N, \quad \int_{\mathbb{R}^N} u^2(x) dx = a^2,$$

where $N \geq 1$, $\alpha \in (0, N)$, $F(s) := \int_0^s f(t) dt$, and $I_\alpha : \mathbb{R}^N \rightarrow \mathbb{R}$ is the Riesz potential. If f is odd in u then we prove the existence of infinitely many normalized solutions. This is joint work with T. Bartsch and Yanyan Liu.

The number and location of critical points of positive solutions of nonlinear elliptic equations

罗鹏, 华中师范大学

This talk is concerning with the property of critical points of positive solutions to the following nonlinear elliptic equations:

$$\begin{cases} -\Delta u = f(u) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^N$ with $N \geq 2$ is a smooth bounded domain and f is a smooth nonlinearity. Using the property of Green's function and degree theory, we give the number and location of critical points of positive solutions under some conditions on Ω and f . This is a work jointed with Prof. Massimo Grossi.

Existence and multiplicity of solutions for logarithmic Schrödinger equations with potential

帅伟, 华中师范大学

We discuss the following logarithmic Schrödinger equation

$$-\Delta u + V(x)u = Q(x)u \log u^2, \quad x \in \mathbb{R}^N.$$

It is known that the corresponding functional is not well defined in $H^1(\mathbb{R}^N)$. By imposing some condition on $V(x)$, we can show that the functional is well defined in a subspace of $H^1(\mathbb{R}^N)$. Then, the existence and multiplicity of solutions is obtained by using variational methods. We remark that the existence of solutions is deeply influenced by the sign of $Q(x)$.

Coupled nonlinear Schrödinger equations with mixed coupling

王志强, 美国Utah州立大学

We discuss results on existence and qualitative property of positive solutions for coupled nonlinear Schrödinger equations with mixed coupling. We discuss the effect of mixed coupling with which coexistence of synchronization and segregation may occur. We examine the asymptotic behavior of these so-

lutions with large coupling, symmetry breaking, and symmetric patterns etc.

On the steady Prandtl boundary layer expansions

张立群, 中科院数学与系统科学研究院

We consider the zero-viscosity limit of the 2D steady Navier-Stokes equations in $(0, L) \times \mathbb{R}^+$ with non-slip boundary conditions. By estimating the stream-function of the remainder, we justify the validity of the Prandtl boundary layer expansion in shear Euler flow with some monotonicity assumptions on the solution of Prandtl's systems and some non-shear Euler flow cases. This is a jointed work with Gao Chen.

Existence of solutions for Schrödinger systems with linear and nonlinear couplings

张志涛, 中科院数学与系统科学研究院

We mainly introduce phase-separation for Schrodinger systems with nonlinear couplings; then introduce some new results on the existence and bifurcations for Schrodinger systems with linear and nonlinear couplings.

Eigenvalue problem for a p-Laplacian equation with trapping potentials

周焕松, 武汉理工大学

本报告将主要介绍应用约束变分极小化和能量估计的方法来研究这类特征值问题解的存在性以及关于相关参数的渐近行为.

Positive least energy solutions for k -coupled Schrödinger system with critical exponent: the higher dimension and cooperative case

邹文明, 清华大学

In this paper, we study the following k -coupled nonlinear Schrödinger system with Sobolev critical exponent:

$$\begin{cases} -\Delta u_i + \lambda_i u_i = \mu_i u_i^{2^*-1} + \sum_{j=1, j \neq i}^k \beta_{ij} u_i^{\frac{2^*}{2}-1} u_j^{\frac{2^*}{2}} & \text{in } \Omega, \\ u_i > 0 & \text{in } \Omega \quad \text{and} \quad u_i = 0 \quad \text{on } \partial\Omega, \quad i = 1, 2, \dots, k. \end{cases}$$

Here $\Omega \subset \mathbb{R}^N$ is a smooth bounded domain, $2^* = \frac{2N}{N-2}$ is the Sobolev critical exponent, $-\lambda_1(\Omega) < \lambda_i < 0, \mu_i > 0$ and $\beta_{ij} = \beta_{ji} \neq 0$, where $\lambda_1(\Omega)$ is the first eigenvalue of $-\Delta$ with the Dirichlet boundary condition. We prove the existence of the positive least energy solution of the k -coupled system for the purely cooperative case $\beta_{ij} > 0$, in higher dimension $N \geq 5$. Since the k -coupled case is much more delicated, we shall introduce the idea of induction.
